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Welcome

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A Newton-type Method for Non-smooth Multi-objective Optimization

Titus Pinta, Sorin-Mihai Grad

Main Result

Given $F : \mathbb{R}^m \to \mathbb{R}^n$, $F(x) = [f_1(x), \ldots, f_n(x)]$, with Newton differentiable Jacobian (e.g. piecewise $\mathcal{C}^2$)

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The Newton-type algorithm converges superlinearly

Outline

1. What is argmin^*?

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2. Under-determined Systems of Equations

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What is argmin^*?

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No total order on $\mathbb{R}^m$

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Let $\sigma \in \Delta^n$, $G(x) = \sigma^T \nabla F(x)$ (first order scalarization)
NOT good : $F(x) = [x_1^2 - x_2, x_2]$ only one good guess for σ

[a] $\Delta^n = \{x \in [0,1]^n \mid \mathbf{1}^T x = 1\}$

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First Order Equation

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$$G(\bar{x}, \bar{\sigma}) = 0 \tag{FOE}$$

Under-determined Systems of Equations

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Newton's method: linearize (UDE) and solve

$$\mathcal{A}(x) = \{y \in \mathbb{R}^m \mid G(x) + \nabla G(x)^T (y - x) = 0\} \qquad \text{(EUDE)}$$

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$\mathcal{A}(x)$ is **not** a singelton

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We can project on $\mathcal{A}(x)$

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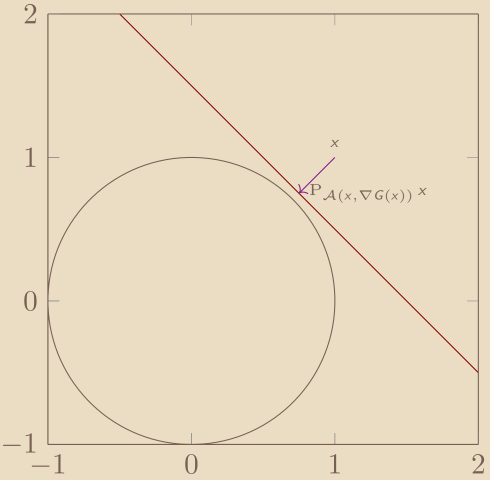
First order optimality ($\nabla G(x^k)$ full rank)

$$x^{k+1} = x^k - \nabla G(x^k)^{+^a} G(x^k)$$

Under-determined Equations



Under-determined Systems of Equations

Newton Differentiability

Definition
$G : \mathbb{R}^m \to \mathbb{R}^n$ is *pointwise Newton differentiable at $\bar{x}$* if there is
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Newton Differentiability

Definition

$G : \mathbb{R}^m \to \mathbb{R}^n$ is *uniformly Newton differentiable on V* if there is $\mathcal{H}G : \mathbb{R}^m \rightrightarrows \mathbb{R}^{n\times m}$ such that $\forall \varepsilon > 0\ \exists \delta$, $\forall x \in \mathbb{R}^m$, $\forall y \in V$ with $\|x - y\| \leq \delta$,

$$\sup_{H\in\mathcal{H}G(x)} \frac{\|G(x) - G(y) - H(x - y)\|}{\|x - y\|} < \varepsilon$$

Examples

Proposition

$G : \mathbb{R}^m \to \mathbb{R}^n$, K compact, $G \in \mathcal{C}^1(K)$, then G is uniformly Newton differentiable on K with a Newton differential $\mathcal{H}(x) := \{\nabla G(x)^T\}$

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$G : \mathbb{R}^m \to \mathbb{R}^n$ is Lipschitz *uniformly semi-smooth* on V* if $\forall \varepsilon > 0 \; \exists \delta$ such that $\forall x$, $\forall y \in V$ with $\|x - y\| \leq \delta$,

$$\frac{\|F'(x; x - y) - F'(y; x - y)\|}{\|x - y\|} \leq \varepsilon$$

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$G:\mathbb{R}^m \to \mathbb{R}^n$, G uniformly semi-smooth* on V, then G is uniformly Newton differentiable on V with a Newton differential $\mathcal{H}(x) := \overline{\mathrm{conv}}\left\{H \in \mathbb{R}^{n\times m} \mid \exists \{x^k\}_{k\in\mathbb{N}}, \lim_{k\to\mathbb{N}} \nabla G(x^k)^T = H\right\}$

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Non-smooth Newton

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Algorithm: $\mathcal{N} : \mathbb{R}^m \rightrightarrows \mathbb{R}^m$,

$$\mathcal{N}(x) = \{x - H^+ G(x) \mid H \in \mathcal{H}G(x)\}$$

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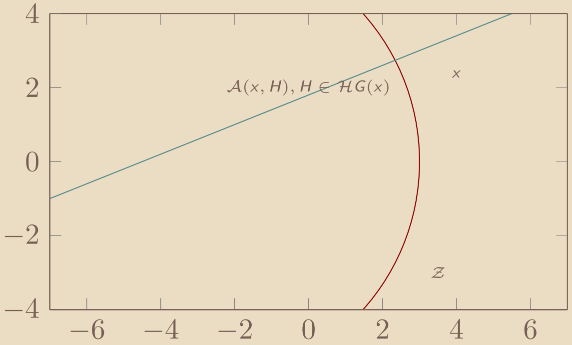
Geometrically Compatible

Definition

$\mathcal{H}G$ is *geometrically compatible* if

$$\|\mathrm{P}_{\mathcal{A}(x,H)}\,x - \mathrm{P}_{\mathcal{Z}}\,\mathrm{P}_{\mathcal{A}(x,H)}\,x\| \leq P\|\mathrm{P}_{\mathcal{Z}}\,\mathrm{P}_{\mathcal{A}(x,H)}\,x - \mathrm{P}_{\mathcal{A}(x,H)}\,\mathrm{P}_{\mathcal{Z}}\,\mathrm{P}_{\mathcal{A}(x,H)}\,x\|$$



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Theorem

If $\mathcal{H}G$ is single-valued and uniformly continuous then it is geometrically compatible

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Using non-expansiveness of P_K, $y \in \mathcal{N}(x)$

$$\mathrm{dist}(\mathrm{P}_K\, y, \mathcal{K}) \leq \| \mathrm{P}_K\, y - \mathrm{P}_{\mathcal{K}}\, x\| \leq \|y - \mathrm{P}_{\mathcal{K}}\, x\| \leq c\|x - \mathrm{P}_{\mathcal{K}}\, x\|$$

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Back to Multi-objective

Problem

Find, $\bar{x} \in \mathbb{R}^m, \bar{\sigma} \in \Delta^n$

$$\bar{\sigma}^T \nabla F(\bar{x}) = 0 \qquad \text{(FOE)}$$

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Assume: ∇F is uniformly Newton differentiable on $\mathcal{Z}$ with $\mathcal{H}F$

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Algorithm: $H^k \in \mathcal{H}F(x^k)$

$$\begin{bmatrix} x^{k+1} \\ \sigma^{k+1} \end{bmatrix} = \mathrm{P}_K\left(\begin{bmatrix} x^k \\ \sigma^k \end{bmatrix} - \begin{bmatrix} {\sigma^k}^T H^k & \nabla F(x^k) \end{bmatrix}^+ {\sigma^k}^T \nabla F(x^k)\right)$$

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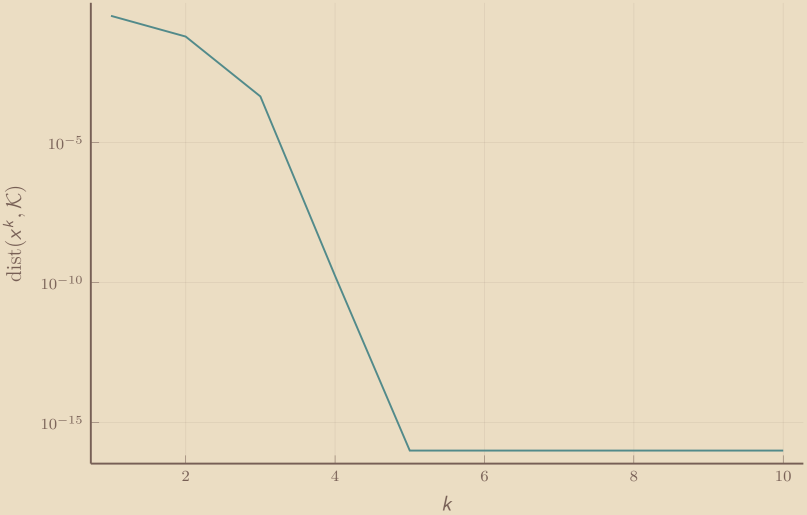
$$\lim_{k \to \infty} \frac{\mathrm{dist}(x^{k+1}, \mathcal{Z} \cap \mathbb{R}^m \times \Delta^n)}{\mathrm{dist}(x^k, \mathcal{Z} \cap \mathbb{R}^m \times \Delta^n)} = 0$$

Numerical Example

$$F(x) = [x_1^2 - x_2, x_2], \quad \mathcal{K} = \{0\} \times \mathbb{R} \times \{.5\} \times \{.5\}$$



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Celebrating the 50th Anniversary of EURO

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